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A Decisive Test of Superstring-Inspired $E(6)$ Models

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Abstract

We point out that in a large class of superstring-inspired $E(6)$ models, either an e^+e^- collider operating at a center-of-mass energy $\sqrt{s} = 1.5$ TeV or higher must detect the pair production of charged or neutral exotic leptons, or an e^+e^- collider with $\sqrt{s} \geq 300$ GeV must discover at least one light neutral Higgs boson with invisible branching ratio exceeding 50%. If neither of these two signals is seen, the lightest neutral exotic lepton would overclose the universe, and the model could be completely excluded, independent of the values of the numerous free parameters. Future Higgs searches might lower the energy of the e^+e^- collider needed to test these models decisively. The only assumptions we have to make are that R -parity is exact, so that the lightest exotic lepton is stable if it is lighter than the lightest neutralino, and that no $SO(10)$ singlet scalar gets a vacuum expectation of order 10^{10} GeV or higher. If the second condition is violated, the model effectively reduces to an $SO(10)$ model as far as collider experiments are concerned.

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1) Introduction

Superstring-inspired $E(6)$ models [1] contain a large number of particles in addition to those present in the Standard Model (SM): Superpartners of the known matter fermions and gauge bosons; scalar di- or lepto-quarks; extended gauge and Higgs sectors; and new “exotic” quarks and leptons. Indeed, this class of models can almost be considered to be a maximal (weakly interacting) extension of the SM. It is this aspect, rather than the by now quite tenuous connection to superstring theory [2], that keeps interest in these models alive [3].

Unfortunately most of the new particles predicted by $E(6)$ models could be very heavy. In the absence of a comprehensive theory of supersymmetry breaking we are not able to give firm upper bounds on sparticle masses [4]. The masses of the new gauge bosons contained in such models can be made very large by postulating large vacuum expectation values (vevs) for Higgs fields that are singlets under the SM gauge group. Finally, most of the exotic fermions reside in vector-like representations of the SM gauge group; they can therefore also be made very heavy. In the simplest of these models, where one requires gauge symmetry breaking to be triggered by radiative corrections [5, 6], these possibly large scales are in fact all related: The vevs of most Higgs singlets cannot exceed the SUSY breaking scale significantly, and the same vevs also give rise to the masses of the exotic leptons, with Yukawa couplings of order 1 or less if the theory is to remain weakly interacting all the way up to the scale of grand unification (GUT). This allows to derive non-trivial relations [6] between some of these masses, but unfortunately does not exclude the possibility that they are all very large.

There are exceptions to this rule, however. Each fermion generation of $E(6)$ contains 27 degrees of freedom. Two of those are SM singlets, commonly called ν_R and N . The ν_R resides in the **16** of $SO(10)$, together with the 15 degrees of freedom that form a complete fermion generation in the SM. The ν_R fields might be exactly massless; alternatively, they might get very large masses through non-renormalizable operators if some scalar $\tilde{\nu}_R$ field gets a vev at an “intermediate” scale around 10^{10} GeV or more, which could also give rise to see-saw type neutrino mass matrices [7]. Either way it is very difficult to derive significant constraints on these ν_R fields.[†]

In contrast, the N superfields are singlets under $SO(10)$. Their fermionic components can acquire masses at the weak scale only by mixing with the neutral components of exotic $SU(2)$ doublet fermions [9]; there are no terms of the type N^3 in the superpotential, since N is not a singlet under the complete gauge group. Note that the vevs that give rise to this mixing also break the $SU(2) \times U(1)_Y$ symmetry of the SM, which means that they contribute to the masses of the W and Z bosons. This allows to derive a firm upper bound [10] of just over 100 GeV for the mass of the lightest eigenstate resulting from this mixing, if the relevant Yukawa couplings are required not to have a Landau pole below the GUT scale. This bound can only be avoided if some N scalar has a vev of order 10^{10} GeV or more, and if the superfields containing the light exotics have non-renormalizable couplings to this vev. However, the same vev would also allow to give very large masses to all new gauge bosons

[†]Successful nucleosynthesis in the early universe requires a large freeze-out temperature, i.e. large masses for gauge bosons coupling to ν_R , if ν_R is exactly massless [8]; however, direct experimental searches [3] by now constrain these gauge bosons to be heavier than several hundred GeV anyway.

and charged new matter fermions; at scales below $\langle N \rangle$ one would then just end up with the minimal supersymmetric standard model (MSSM) [4]. We will therefore assume that N scalars can only get vevs of the order of the weak or SUSY breaking scales, i.e. a few TeV or less; in this case the bound of ref.[10] holds.

The existence of an upper bound on the mass of an exotic fermion does not yet mean that we can test this model decisively, however. To begin with, present e^+e^- colliders do not have sufficient energy to produce even the lightest exotics for all combinations of parameters. This will change as soon as the next generation of linear e^+e^- colliders goes into operation. However, even here the neutral exotics might be impossible to find. We will argue that the lightest neutral exotic lepton is likely to be stable. In this case it will only give rise to an observable signal at colliders if it is produced in association with a heavier exotic. Unfortunately *no* upper bound on the masses of these heavier states can as yet be given.

In this seemingly hopeless situation cosmological arguments come to the rescue. It turns out that the upper bound on the mass of the lightest exotic *decreases* as the experimental lower bounds on the masses of the heavier exotics increase. Moreover, in this situation the light exotics are forced to have smaller and smaller $SU(2)$ doublet components, which suppresses their couplings to gauge and Higgs bosons. Since the light exotics are stable, some fraction of such particles produced in the very early universe is still around today [11]. This fraction, and hence the contributions of such exotic Big Bang relics to the present matter density, is (approximately) inversely proportional to the annihilation cross section of the exotics. The crucial observation is that light, singlet-dominated exotics will have small annihilation cross sections by virtue of their suppressed couplings, and hence large relic densities. It is therefore clear that at some point the lower experimental bound on the masses of the *heavier* exotic leptons can force the cosmological relic density of the *light* exotic to become unacceptably large. We find that, before this happens, light neutral exotics will become accessible to Higgs boson decays, leading to a large invisible branching ratio for at least one light Higgs boson. It is the purpose of the present paper to study these connections quantitatively. Of course, the heavier exotics might well be discovered at rather low masses, in which case the relic density constraint could be fulfilled easily. In this optimistic scenario one can study the properties of the new fermions in order to further test the model. We wish to emphasize here that at some future time, the combination of collider searches and the relic density constraint will test the model *decisively*, independent of the values of the (many) free parameters. Unfortunately we find that a 500 GeV e^+e^- collider is not quite sufficient to probe the entire parameter space; one will have to push the energy to 1 or even 1.5 TeV in order to close the last loophole. Nevertheless we find it remarkable that a decisive test is possible at all; after all, most experimental bounds can be evaded by simply increasing the “new physics” scale.

The remainder of this paper is organized as follows. In Sec. 2 we describe those parts of the model that are of relevance to us, i.e. the exotic leptons and Higgs bosons; we also give the couplings of the (neutral) exotics to gauge and Higgs bosons. In Sec. 3 the cross sections for the production of neutral exotics at e^+e^- colliders are listed. In Sec. 4 the calculation of the cosmological relic density of the light neutral exotics is discussed; some care must be taken here, due to the prominent role played by s -channel exchange diagrams, as well as the presence of a second light exotic, which can suppress the relic density by co-annihilation. Sec. 5 contains our numerical results, and Sec. 6 is devoted to a brief summary.

2) The Model

For definiteness we will work in the framework of the minimal rank 5 subgroup of $E(6)$ [1], which does not require the introduction of an intermediate scale between the GUT and weak scales. However, our analysis would go through with only minor modifications in models with a gauge group of rank 6 at the weak scale, as well as in models where some $\tilde{\nu}_R$ field has a vev around 10^{10} GeV or more. As already stated in the Introduction, the only important assumption we have to make is that none of the scalar N fields gets a vev much above 10^9 GeV.

In $E(6)$ models each **27**-dimensional representation contains one N superfield, which is an SM singlet, as well as the exotic $SU(2)$ doublet superfields H and \bar{H} , whose scalar components have just the right quantum numbers to serve as the Higgs bosons of the MSSM, i.e. to provide masses for the W and Z bosons. At least one of the N scalars also has to get a vev in order to give a sufficiently large mass to the single new Z' boson of the rank-5 model. Using rotations in generation space, we can always work in a basis where only one H^0 , one \bar{H}^0 and one N field have non-vanishing vev; following the notation of ref.[9] we call these true Higgs fields H_3^0 , \bar{H}_3^0 and N_3 . Their fermionic superpartners then mix with the superpartners of the three neutral gauge bosons to form the six neutralino states of this model [9].

Here we are interested in the first two generations of fermionic H , \bar{H} and N fields. They obtain their masses from the superpotential

$$W_{\text{lep}} = \sum_{i,j,k=1}^3 \lambda_{ijk} H_i \bar{H}_j N_k. \quad (1)$$

In order to give masses to all charged exotic leptons*, we at least have to allow those couplings where exactly one of the indices i, j, k in eq.(1) equals 3, the other two being either 1 or 2. By further rotations between fields of the first two generations we can define $\lambda_{123} = \lambda_{213} = 0$, without loss of generality; the contribution of these couplings to the mass matrix of the charged exotic fermions is then diagonal, with

$$m_{L_1^\pm} = \lambda_{113} x; \quad (2a)$$

$$m_{L_2^\pm} = \lambda_{223} x, \quad (2b)$$

where we have introduced $x \equiv \langle N_3 \rangle$.

In the basis $(\widetilde{H}_1^0, \widetilde{H}_1^0, \widetilde{N}_1, \widetilde{H}_2^0, \widetilde{H}_2^0, \widetilde{N}_2)$, the contribution of these couplings to the mass matrix of the neutral exotic leptons reads:

$$\mathcal{M}_{L^0} = \begin{pmatrix} 0 & m_{L_1^\pm} & \lambda_{131}\bar{v} & 0 & 0 & \lambda_{132}\bar{v} \\ m_{L_1^\pm} & 0 & \lambda_{311}v & 0 & 0 & \lambda_{312}v \\ \lambda_{131}\bar{v} & \lambda_{311}v & 0 & \lambda_{231}\bar{v} & \lambda_{321}v & 0 \\ 0 & 0 & \lambda_{231}\bar{v} & 0 & m_{L_2^\pm} & \lambda_{232}\bar{v} \\ 0 & 0 & \lambda_{321}v & m_{L_2^\pm} & 0 & \lambda_{322}v \\ \lambda_{132}\bar{v} & \lambda_{312}v & 0 & \lambda_{232}\bar{v} & \lambda_{322}v & 0 \end{pmatrix}, \quad (3)$$

*We call these fields “leptons” merely in order to indicate that they do not have strong interactions; they need not carry the same lepton number as the charged leptons and neutrinos of the SM

with $v \equiv \langle H_3^0 \rangle$, $\bar{v} \equiv \langle \bar{H}_3^0 \rangle$. The neutral exotic leptons are Majorana fermions; the mass matrix (3) is therefore symmetric.

The coupling λ_{333} in eq.(1) must also be non-zero, in order to avoid the presence of a dangerous axion (see below). Terms where none of the three subscripts equals 3 do not contribute to the mass matrices. Finally, there could be contributions to the superpotential (1) where only one of the three indices is not equal to 3. In this case the charged exotic leptons would mix with the charginos, and the neutral exotic leptons would mix with the neutralinos. However, if such couplings are present, the rotations in field space that define the basis where only H_3^0 , \bar{H}_3^0 and N_3 have non-zero vevs would depend on the renormalization scale. Worse, all H_i and \bar{H}_i would then couple to SM quarks and leptons at least at the one loop level, which could lead to dangerous flavour changing neutral currents (FCNC) [12]. We therefore forbid these terms. Fortunately, this is not only technically natural, but even follows automatically if one requires all potentially dangerous terms in the low energy superpotential (which can lead to fast proton decay, large neutrino masses, or tree-level FCNC) to be forbidden by a single discrete Z_2 symmetry [13].

This further restriction of the allowed terms in eq.(1) greatly simplifies the calculation, and leads to a much more appealing model, but has no significant impact on the properties that are of interest to us. This can be seen from the fact that even if all terms in eq.(1) were present, in the limit $v, \bar{v} \rightarrow 0$ the 12×12 neutral fermion mass matrix would have two zero eigenvalues, corresponding to SM singlet fermions $\widetilde{N}_1, \widetilde{N}_2$. Since all couplings in eq.(1) are required to be of order 1 or less [6, 10], this observation implies that even in the realistic situation with non-vanishing v and \bar{v} , there will be two neutral Majorana fermions whose masses are roughly of order M_Z or less. Moreover, the upper bound on the masses of these neutral fermions will decrease as the (experimental lower bounds on the) masses of the charged exotic fermions is increased. We repeat, these crucial properties of the model follow from eq.(1) without further assumptions. For the remainder of our analysis we will concentrate on the simpler case where the mass matrix (3) is decoupled from the neutralino mass matrix, just to avoid needless complications.

A little calculation shows that the determinant of the matrix (3) is proportional to $(v\bar{v}x)^2$; this means that all three vevs have to be non-zero if all eigenvalues are to be non-vanishing. It is also quite easy to see that in the limit $m_{L_{1,2}^\pm} \gg v, \bar{v}$, and with all couplings $\mathcal{O}(1)$ or less, there will be four large eigenvalues, approximately equal to $\pm m_{L_1^\pm}$ and $\pm m_{L_2^\pm}$. The other two eigenvalues are [9] then of order $\lambda^2 v \bar{v} / m_{L_{1,2}^\pm}$. This is why the upper bound on the smaller eigenvalues decreases with increasing mass of the charged exotics, as stated above. Indeed, in ref.[10] it was shown that the mass $m_{L_1^0}$ of the lightest neutral exotic lepton is maximized if all entries of the mass matrix (3) are of the same order (or zero). Numerically one has $m_{L_1^0} \leq 110$ GeV if $m_{L_{1,2}^\pm} \geq 45$ GeV.

In general the matrix (3) has to be diagonalized numerically. The resulting eigenstates L_i^0 are given by

$$L_i^0 = \sum_{j=1}^6 U_{ij} \widetilde{N}_j, \quad (4)$$

where we have introduced the 6-component vector $\widetilde{N} \equiv (\widetilde{H}_1^0, \widetilde{\bar{H}}_1^0, \widetilde{N}_1, \widetilde{H}_2^0, \widetilde{\bar{H}}_2^0, \widetilde{N}_2)$, and U is

an orthogonal[†] matrix chosen such that

$$U\mathcal{M}_{L^0}U^T = \text{diag}(m_{L_i^0}), \quad i = 1, \dots, 6. \quad (5)$$

The couplings of the charged and neutral exotic mass eigenstates to the standard Z boson are then given by the Lagrangean [14]

$$\mathcal{L}_{ZLL} = \frac{g}{2\cos\theta_W} Z_\mu \left[\sum_{a=1}^2 \overline{L}_a^\pm \gamma^\mu L_a^\pm (1 - 2\sin^2\theta_W) + \sum_{i,j=1}^6 \overline{L}_i^0 \gamma^\mu \gamma_5 L_j^0 t_{ij} \right], \quad (6)$$

where g is the $SU(2)$ gauge coupling, θ_W the weak mixing angle, and

$$t_{ij} = \frac{1}{2} (U_{i2}U_{j2} + U_{i5}U_{j5} - U_{i1}U_{j1} - U_{i4}U_{j4}). \quad (7)$$

A few comments are in order here. First, when writing eqs.(5),(6) we have allowed the masses of the neutral exotic leptons to have either sign. We could also insist that all $m_{L_i^0} \geq 0$ by inserting appropriate factors of i in the matrix U , which would then no longer be orthogonal (but would still be unitary, of course). In this case the diagonal couplings of the Z boson to neutral exotics would still be purely axial vector, but the off-diagonal couplings would become purely vector if the two corresponding eigenvalues have opposite signs. Secondly, recall that the L_i^0 are (4-component) Majorana spinors. This means that their couplings appearing in Feynman diagrams are twice as large as those in the Lagrangean (6). Finally, following ref.[9], and contrary to the usual MSSM notation, we have defined $H_{1,2,3}$ to have hypercharge $+1/2$, while the hypercharge of $\bar{H}_{1,2,3}$ is $-1/2$; this explains the overall sign of the coupling t_{ij} in eq.(7).

In our calculation of cosmological relic densities we also have to specify the Higgs sector of the model. Recall that we are working in a basis where only H_3^0 , \bar{H}_3^0 and N_3 have non-zero vevs. The relevant part of the Higgs potential is then given by [15]

$$\begin{aligned} V_{\text{Higgs}} = & m_{H_3}^2 |H_3^0|^2 + m_{\bar{H}_3}^2 |\bar{H}_3^0|^2 + m_{N_3}^2 |N_3|^2 + (\lambda_{333} A_{333} H_3^0 \bar{H}_3^0 N_3 + h.c.) \\ & + \lambda_{333}^2 \left[|H_3^0 \bar{H}_3^0|^2 + |N_3|^2 (|H_3^0|^2 + |\bar{H}_3^0|^2) \right] \\ & + \frac{g'^2}{8\cos^2\theta_W} \left(|H_3^0|^2 - |\bar{H}_3^0|^2 \right)^2 + \frac{g'^2}{72} \left(5|N_3|^2 - 4|\bar{H}_3^0|^2 - |H_3^0|^2 \right)^2, \end{aligned} \quad (8)$$

where g' is the $U(1)$ gauge coupling. We will assume the Higgs mass parameters as well as the soft breaking parameter A_{333} to be real. The vevs can then all be chosen to be real [16], and the mass matrices for the real (scalar) and imaginary (pseudoscalar) parts of the neutral Higgs fields decouple. The former is given by [9]

$$\mathcal{M}_H^2 = \begin{pmatrix} \left(\frac{g^2}{2} + \frac{25}{18}g'^2 \right) v^2 - \tilde{A} \frac{\bar{v}x}{v} & \left(2\lambda_{333}^2 - \frac{g^2}{2} - \frac{5}{18}g'^2 \right) v\bar{v} + \tilde{A}x & \left(2\lambda_{333}^2 - \frac{10}{9}g'^2 \right) vx + \tilde{A}\bar{v} \\ \left(2\lambda_{333}^2 - \frac{g^2}{2} - \frac{5}{18}g'^2 \right) v\bar{v} + \tilde{A}x & \left(\frac{g^2}{2} + \frac{5}{9}g'^2 \right) \bar{v}^2 - \tilde{A} \frac{vx}{\bar{v}} & \left(2\lambda_{333}^2 - \frac{5}{18}g'^2 \right) \bar{v}x + \tilde{A}v \\ \left(2\lambda_{333}^2 - \frac{10}{9}g'^2 \right) vx + \tilde{A}\bar{v} & \left(2\lambda_{333}^2 - \frac{5}{18}g'^2 \right) \bar{v}x + \tilde{A}v & \frac{25}{18}g'^2 x^2 - \tilde{A} \frac{v\bar{v}}{x} \end{pmatrix}, \quad (9)$$

[†]We assume that all couplings in eq.(1) are real.

with $\tilde{A} \equiv \lambda_{333} A_{333}$. In writing eq.(9) we have used the requirement that the first derivatives of the potential (8) with respect to the fields should vanish in the minimum. Notice that the smallest eigenvalue $m_{h_1^0}^2$ of the matrix (9) is again only of order $v^2 + \bar{v}^2$, not of order of the Z' boson mass or the SUSY breaking scale [17]:

$$m_{h_1^0}^2 \leq M_Z^2 \left[\cos^2(2\beta) + \frac{2\lambda_{333}^2 \cos^2 \theta_W}{g^2} \sin^2(2\beta) + \frac{\sin^2 \theta_W}{9} (4\sin^2 \beta + \cos^2 \beta)^2 \right], \quad (10)$$

where $\tan \beta \equiv v/\bar{v}$ (> 1). Eqs.(9) and (10) only hold at tree level. There are sizable radiative corrections from top and stop loops, as well as possibly from loops involving exotic (s)quarks and (s)leptons [18]. However, for our purpose their effect can largely be mimicked by allowing λ_{333} at the weak scale to be quite large. The reason is that these corrections tend to increase the mass of the lightest scalar Higgs boson; eq.(10) shows that increasing λ_{333} has the same effect. Moreover, the radiative corrections are only important if they involve large Yukawa couplings, e.g. that of the top quark. However, the introduction of these large couplings also *reduces* the upper bound on λ_{333} which follows from the requirement that the model remains weakly interacting up to the GUT scale. The final upper bound on $m_{h_1^0}$ is therefore not much changed by these corrections.

Since the model contains two massive neutral gauge bosons, only one physical pseudoscalar A survives. It can be written as [9]

$$\begin{aligned} A &= \frac{1}{\sqrt{\bar{v}/v + v/\bar{v} + v\bar{v}/x^2}} \frac{1}{\sqrt{2}} \Im \left(\frac{\bar{v}}{v} H_3^0 + \frac{v}{\bar{v}} \bar{H}_3^0 + \frac{v\bar{v}}{x^2} N_3 \right) \\ &\equiv \frac{1}{\sqrt{2}} \sum_{j=1}^3 P_j \Im (H_j), \end{aligned} \quad (11)$$

where we have introduced the complex 3-component vector $H \equiv (H_3^0, \bar{H}_3^0, N_3)$, and the symbol \Im denotes the imaginary part. The mass of the physical pseudoscalar is given by [9]

$$m_A^2 = -\lambda_{333} A_{333} x \left(\frac{\bar{v}}{v} + \frac{v}{\bar{v}} + \frac{v\bar{v}}{x^2} \right). \quad (12)$$

Notice that this state would be massless if $\lambda_{333} = 0$, as mentioned earlier. There is also a physical charged Higgs boson, with mass [9]

$$m_{H^\pm}^2 = M_W^2 \left(1 - \frac{2\lambda_{333}^2}{g^2} \right) + m_A^2 + \lambda_{333} A_{333} \frac{v\bar{v}}{x}. \quad (13)$$

Note that this Higgs boson can be lighter than the W boson if m_A is small and λ_{333} is sizable.

In general the mass matrix (9) is most easily diagonalized numerically. Since it is real and symmetric, the diagonalization can be achieved by an orthogonal matrix S :

$$S \mathcal{M}_H^2 S^T = \text{diag}(m_{h_1^0}^2, m_{h_2^0}^2, m_{h_3^0}^2). \quad (14)$$

The physical scalars h_i^0 are then given by

$$h_i^0 = \frac{1}{\sqrt{2}} \sum_{j=1}^3 S_{ij} \Re (H_j). \quad (15)$$

The matrix S and the vector P introduced in eq.(11) determine the couplings of the physical Higgs bosons to SM particles. We need the $Z - Z - h_i^0$ couplings in order to interpret bounds on the Higgs sector from searches at LEP; they are given by

$$g_{ZZh_i^0} = \frac{gM_Z}{\cos\theta_W} (S_{i1}\sin\beta + S_{i2}\cos\beta). \quad (16)$$

Bounds on $Z \rightarrow h_i^0 A$ decays involve the couplings

$$g_{ZAhh_i^0} = \frac{g}{2\cos\theta_W} (S_{i1}P_1 - S_{i2}P_2). \quad (17)$$

In Sec. 4 we will also need the couplings of the Higgs bosons to the quarks and leptons of the SM. We write the corresponding Lagrangean as

$$\mathcal{L}_{Hff} = \frac{g}{\cos\theta_W} \bar{f} \left[c^{(f)}(-i\gamma_5)A + \sum_{i=1}^3 d_i^{(f)}h_i^0 \right] f, \quad (18)$$

with

$$c^{(f)} = -\frac{m_f}{2M_Z} \cdot \begin{cases} P_1/\sin\beta, & f = u, c, t \\ P_2/\cos\beta, & f = e, \mu, \tau, d, s, b \end{cases} \quad (19a)$$

$$d_i^{(f)} = -\frac{m_f}{2M_Z} \cdot \begin{cases} S_{i1}/\sin\beta, & f = u, c, t \\ S_{i2}/\cos\beta, & f = e, \mu, \tau, d, s, b \end{cases} \quad (19b)$$

Finally, we will need the couplings of the physical Higgs bosons to the neutral exotic leptons. They can most easily be expressed in terms of the couplings λ'_{ijk} between the lepton and Higgs current eigenstates:

$$\mathcal{L}_{\tilde{N}\tilde{N}H} = -\frac{1}{4} \sum_{i,j=1}^6 \sum_{k=1}^3 \lambda'_{ijk} \overline{\tilde{N}}_i (1 - \gamma_5) \tilde{N}_j H_k^0 + h.c., \quad (20)$$

where \tilde{N} is the 6-component vector of Majorana spinors introduced in eq.(4), and H the 3-component vector of complex neutral Higgs fields defined in eq.(11). The couplings λ'_{ijk} are determined by the superpotential (1):

$$\begin{aligned} \lambda'_{123} &= \lambda_{113}, \quad \lambda'_{132} = \lambda_{131}, \quad \lambda'_{153} = \lambda_{123}, \quad \lambda'_{162} = \lambda_{132}, \quad \lambda'_{231} = \lambda_{311}, \quad \lambda'_{243} = \lambda_{213}, \\ \lambda'_{261} &= \lambda_{312}, \quad \lambda'_{342} = \lambda_{231}, \quad \lambda'_{351} = \lambda_{321}, \quad \lambda'_{453} = \lambda_{223}, \quad \lambda'_{462} = \lambda_{232}, \quad \lambda'_{561} = \lambda_{322}; \end{aligned} \quad (21)$$

this has to be symmetrized in the first two indices ($\lambda'_{ijk} = \lambda'_{jik}$), and all other λ' couplings vanish. Recall that without loss of generality we can work in a basis where the mass matrix for the charged exotic leptons is diagonal, i.e. $\lambda_{123} = \lambda_{213} = 0$; this implies $\lambda'_{153} = \lambda'_{243} = 0$ in this basis. The interaction between lepton and Higgs mass states can then be written as

$$\begin{aligned} \mathcal{L}_{LLH} &= -\frac{1}{2\sqrt{2}} \sum_{i,j=1}^6 \sum_{k=1}^3 \lambda'_{ijk} \sum_{l=1}^3 S_{lk} h_l^0 \sum_{m,n=1}^6 U_{mi} U_{nj} \overline{L}_m^0 L_n^0 \\ &+ \frac{i}{2\sqrt{2}} \sum_{i,j=1}^6 \sum_{k=1}^3 \lambda'_{ijk} P_k A \sum_{m,n=1}^6 U_{mi} U_{nj} \overline{L}_m^0 \gamma_5 L_n^0, \end{aligned} \quad (22)$$

where the orthogonal matrices U and S have been defined in eqs.(5) and (14), respectively, and the eigenvector P of the pseudoscalar mass matrix is given in eq.(11). Recall that the L_m^0 are Majorana fermions, and the physical Higgs bosons h_i^0 and A are described by real fields.

3) Constraints from e^+e^- Colliders

In this section we discuss how new physics searches at existing and future e^+e^- colliders can constrain the class of $E(6)$ models we are considering. At present the most stringent constraints on both the exotic leptons and the Higgs sector come from LEP. To begin with, the failure to observe the charged exotic leptons immediately implies that their masses must exceed 45 GeV, since eq.(6) shows that they always couple with essentially full gauge strength to the Z boson.

The decay of the Z into two neutral exotic leptons needs a somewhat more detailed discussion. The heavier L_i^0 states can always decay into L_1^0 plus a real or virtual Z or Higgs boson. However, the lightest state L_1^0 can only decay into SM particles or the right-handed neutrino state ν_R , or their superpartners. Such decays could be due to terms in the superpotential of the type $\bar{H}_{1,2}l_L e_R$, $\bar{H}_{1,2}q_L d_R$ or $H_{1,2}q_L u_R$, where q_L and l_L are quark and lepton $SU(2)$ doublet superfields and e_R , d_R and u_R the corresponding singlets. However, couplings of this kind will in general generate tree-level FCNC; these terms are therefore severely constrained [19], which is why they are often completely forbidden in (potentially) realistic models, e.g. by means of a discrete symmetry [13]. The model in general also allows couplings of the kind $H_{1,2}l_L \nu_R$. Such couplings can lead to flavor changing processes in the lepton sector ($\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, etc), but only at one-loop level; they are therefore somewhat less tightly constrained [19]. If any of these couplings exists, the exotic leptons become odd under R parity. This means that the decay product of L_1^0 must contain a superparticle. Since L_1^0 is quite light, we can expect that only final states containing the lightest sparticle, which in almost all cases is the lightest neutralino \tilde{Z}_1 , can occur. The least tightly constrained of the above couplings, $H_{1,2}l_L \nu_R$, will then lead to an invisible final state, if we assume that R parity is conserved. Of course, it is not at all clear that such decays are kinematically allowed, since \tilde{Z}_1 could very well be heavier than L_1^0 . Indeed, in view of the fact that present bounds [3, 8] require the Z' boson mass to exceed several hundred GeV, we need a rather high value of the SUSY breaking scale in this model [5, 6]. We therefore conclude that L_1^0 most probably is either absolutely stable, or decays invisibly; in particular, this is always true if tree-level FCNC are suppressed by a simple symmetry and R parity is conserved. In this case L_1^0 cannot be detected directly by collider experiments. Since we want to devise a test of the model that works even under the least favorable circumstances we will assume that L_1^0 is indeed invisible.

The partial width of $Z \rightarrow L_i^0 L_j^0$ decays is given by [14]:

$$\Gamma(Z \rightarrow L_i^0 L_j^0) = \frac{|\vec{k}|}{24\pi M_Z^2} (2 - \delta_{ij}) \left(\frac{gt_{ij}}{\cos\theta_W} \right)^2$$

$$\cdot \left[2M_Z^2 - m_i^2 - m_j^2 - 6m_i m_j - \frac{(m_i^2 - m_j^2)^2}{M_Z^2} \right], \quad (23)$$

where we have used the shorthand notation $m_i \equiv m_{L_i^0}$. The coupling t_{ij} has been defined in eq.(7), and the L_i^0 3-momentum in the Z rest frame $|\vec{k}|$ is given by

$$|\vec{k}| = \frac{1}{2M_Z} \sqrt{(M_Z^2 - m_i^2 - m_j^2)^2 - 4m_i^2 m_j^2}. \quad (24)$$

Recall that in our convention the neutral lepton masses, and hence the bi-linear term in eq.(23), can have either sign.

We have argued above that the combination $i = j = 1$ only contributes to the invisible width of the Z boson. However, even though the heavier neutral leptons will almost always decay inside the detector, their production might not lead to an experimentally observable final state if the mass difference to the lightest lepton is too small. We have quite conservatively required $|m_i| - |m_1| \geq 3$ GeV for an “experimentally visible” L_i^0 . The signature for the production of these visible states is quite similar to that for the production of the heavier neutralino states of the MSSM. Searches [20] for such neutralinos have so far not been successful; the bound on the resulting branching ratio is of the order of (a few times) 10^{-5} , the precise value depending on the masses of the particles involved. We have therefore imposed the upper bound

$$\Gamma(Z \rightarrow \text{visible exotics}) < 0.13 \text{ MeV}, \quad (25)$$

which corresponds to a bound on the branching ratio of about $5 \cdot 10^{-5}$.

The best bound on non-SM contributions to invisible Z decays comes from the measurement of $\Gamma_{\text{inv}}(Z)/\Gamma(Z \rightarrow l^+ l^-)$; this can be interpreted [21] as a measurement of the number of SM neutrino species, $N_\nu = 2.988 \pm 0.023$. Taking into account that the model predicts $N_\nu \geq 3$, this translates into the bound

$$\Delta\Gamma_{\text{inv}} \leq 6.7 \text{ MeV} \quad (26)$$

at 95% confidence level. Note that “invisible” includes everything that is not counted in any explicitly reconstructed final state; in particular, $L_i^0 L_j^0$ final states where neither of the two leptons is at least 3 GeV heavier than L_1^0 are included here.

At future high energy $e^+ e^-$ colliders the neutral exotic leptons can be produced via the exchange of a virtual Z boson. The cross section is given by

$$\begin{aligned} \sigma(e^+ e^- \rightarrow L_i^0 L_j^0) &= \frac{|\vec{k}|}{4\pi\sqrt{s}} (2 - \delta_{ij}) \left(\frac{g}{\cos\theta_W} \right)^4 \frac{(v_e^2 + a_e^2) t_{ij}^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \\ &\cdot \left(E_i E_j - m_i m_j + \frac{1}{3} |\vec{k}|^2 \right), \end{aligned} \quad (27)$$

with $a_e = -1/4$ and $v_e = 1/4 - \sin^2 \theta_W$. $|\vec{k}|$ is given by eq.(24) with M_Z replaced by the center-of-mass energy \sqrt{s} , and $E_i = \sqrt{m_i^2 + |\vec{k}|^2}$. In principle this cross section also receives contributions from Z' boson exchange. However, if these contributions are sizable,

Z' exchange will also lead to observable effects in the pair production of SM fermions. Future e^+e^- colliders will therefore be able [2] to find evidence for a Z' boson with mass up to several times \sqrt{s} . The discovery of a Z' signal would not only (obviously) rule out the SM, but would also allow significant new tests of the type of model we are studying here. In order to be conservative we therefore always assume that the contributions from Z' exchange are too small to be detectable. Finally, we will always assume that future e^+e^- colliders will probe for the existence of charged exotic leptons almost up to the kinematical limit, since they have full strength couplings to both the Z boson and the photon.

The Higgs sector of the model is also constrained by unsuccessful new physics searches at LEP. We have used the results of the ALEPH collaboration [22], which has published bounds on the ZZh_i^0 and ZAh_i^0 couplings of light Higgs bosons. Their numerical bounds can be approximated by:

$$(S_{i1} \sin\beta + S_{i2} \cos\beta)^2 \leq \begin{cases} 0.025, & m_{h_i^0} \leq 10 \text{ GeV} \\ 0.005m_{h_i^0} - 0.025, & 10 \text{ GeV} \leq m_{h_i^0} \leq 20 \text{ GeV} \\ 0.0175m_{h_i^0} - 0.275, & 20 \text{ GeV} \leq m_{h_i^0} \leq 30 \text{ GeV} \\ 0.025m_{h_i^0} - 0.5, & 30 \text{ GeV} \leq m_{h_i^0} \leq 60 \text{ GeV} \end{cases} \quad (28a)$$

$$(S_{i1}P_1 - S_{i2}P_2)^2 \leq \begin{cases} 0.1, & m_{h_i^0} + m_A \leq 81 \text{ GeV} \\ 0.1(m_{h_i^0} + m_A - 81 \text{ GeV}) + 0.1, & m_{h_i^0} + m_A \leq M_Z \end{cases} \quad (28b)$$

where all masses are in GeV.

As emphasized in Sec. 2, at least one of the three neutral scalar Higgs bosons of the model must have mass below 150 GeV or so; moreover, this boson will couple to the Z with full strength if the bound (10) on its mass is saturated. For some combinations of parameters the lightest Higgs scalar will have a very weak coupling to the Z ; however, in this case the next-to-lightest Higgs scalar will have unsuppressed coupling to the Z , and its mass will also satisfy the bound (10). In fact, the scalar Higgs sector of our model is quite similar to that of the MSSM with additional Higgs singlet superfield, where it has been shown [23] that an e^+e^- collider with $\sqrt{s} \geq 300$ GeV has to detect at least one neutral scalar Higgs boson. Unfortunately this may not be sufficient to allow a significant test of the model, since this Higgs boson might look very similar to the single Higgs boson of the SM. Even if several Higgs bosons are discovered, it might be quite difficult to distinguish between the $E(6)$ model we are discussing here and the MSSM, since large x means that one of the three neutral Higgs scalars is quite heavy and essentially a pure singlet; the singlet components of the other two scalar Higgs bosons and the single pseudoscalar Higgs boson are then very small. We therefore try to avoid making assumptions about searches for neutral Higgs bosons at future colliders as much as possible. There is one exception, however: The light Higgs boson(s) of the model might have large branching ratios into neutral exotic leptons, which could lead to a large invisible branching ratio for these Higgs particles. This would be fairly distinctive; in the MSSM a large invisible branching ratio of a Higgs boson would imply that the light chargino will be discovered at LEP2 [24]. In contrast, we will always require that the charged exotic leptons are too heavy to be produced at the e^+e^- collider under consideration.

The partial widths for Higgs decays into exotic lepton can be written as

$$\Gamma(H \rightarrow L_i^0 L_j^0) = \frac{\lambda^2}{4\pi m_H^3} (2 - \delta_{ij}) \sqrt{m_H^2 - (m_i + m_j)^2} \sqrt{m_H^2 - (m_i - m_j)^2} \cdot [m_H^2 - (m_i \pm m_j)^2]. \quad (29)$$

As usual, the lepton masses $m_{i,j}$ can have either sign here. The “+” (“−”) sign in the last term of eq.(29) applies if H is scalar (pseudoscalar), and λ is the relevant $HL_i^0 L_j^0$ coupling in the Lagrangean (22); recall that a factor of 2λ appears in the relevant Feynman rule, due to the Majorana nature of L_i^0 . When computing the total decay widths of the neutral Higgs bosons of our model, we include decays into SM fermion pairs as well as (for the scalar Higgs bosons) W^+W^- and ZZ final states, where we allow one (but not both) of the gauge bosons to be off-shell. Expressions for the relevant partial widths can, e.g., be found in the recent review article [24]; of course, we have to use the couplings listed in eqs.(16)–(19) here. Note that the quark masses in eqs.(19) are meant to be running masses, taken at scale m_H ; we also include the leading non-logarithmic QCD corrections [25] (which remain finite in the limit $m_q/m_H \rightarrow 0$) to the $c\bar{c}$ and $b\bar{b}$ partial widths. An accurate estimate of the decay widths of the light Higgs bosons of the model is necessary not only for the evaluation of branching ratios for exotic Higgs decays, but also for the calculation of the cosmological relic density, to which we turn next.

4) The Cosmological Relic Density

We argued in the previous section that the exotic leptons will have odd R parity if they have any Yukawa interactions with ordinary matter. The lightest exotic lepton L_1^0 will then be absolutely stable if it is lighter than the lightest neutralino \tilde{Z}_1 . The only way out would be to introduce R parity breaking interactions in the superpotential. However, in this case the model, although very complicated, would no longer be able to explain the existence of cold dark matter (CDM); this seems to be required [26] for a successful fit of all data on large scale structure in the universe. The fact that \tilde{Z}_1 makes an excellent CDM candidate is often viewed as one of the strengths of the MSSM; a significantly more complicated model that gives up this advantage does not look very appealing to us. We will therefore require R parity to be conserved.

Unfortunately the usual approximation [11] for the calculation of the relic density is often not reliable in this model. Two of the three cases discussed in ref.[27] as examples where the usual method is not applicable can occur here, possibly simultaneously: Narrow s –channel (Higgs) poles play a very important role in the calculation of the annihilation cross section; and the next-to-lightest neutral exotic L_2^0 can be quite close in mass to L_1^0 , in which case $L_1^0 L_2^0$ co-annihilation as well as $L_2^0 L_2^0$ annihilation have to be considered as well.

We have followed ref.[27] in our computation of the relic density. For completeness we list, but do not derive, the relevant expressions here. The calculation proceeds in two steps. First one has to determine the freeze-out temperature T_f , where the relic particles fall out of thermal equilibrium. It is given by

$$x_f = \ln \frac{0.038 g_{\text{eff}} M_P |m_1| \langle \sigma_{\text{eff}} v \rangle (x_f)}{\sqrt{g_* x_f}}, \quad (30)$$

where we have introduced $x_f \equiv |m_1|/T_f$, m_1 being the mass of the lightest neutral exotic. $M_P = 1.22 \cdot 10^{19}$ GeV is the Planck mass, and g_* is the number of degrees of freedom that are in thermal equilibrium at temperature T_f ; we have for simplicity used a fixed $\sqrt{g_*} = 9$, which introduces a negligible error in our calculation. Eq.(30) has to be solved iteratively, since the r.h.s. depends on x_f both explicitly and via the thermal average $\langle \sigma_{\text{eff}} v \rangle$, defined as

$$\langle \sigma_{\text{eff}} v \rangle(x) = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv v^2 e^{-v^2 x/4} \sigma_{\text{eff}} v, \quad (31)$$

where v is the relative velocity of the two annihilating particles in their center-of-mass frame. In the presence of co-annihilation, the effective annihilation cross section σ_{eff} is given by

$$\sigma_{\text{eff}}(x) = \frac{4}{g_{\text{eff}}^2} \left[\sigma_{11} + 2\sigma_{12}(1 + \Delta)^{3/2} e^{-x\Delta} + \sigma_{22}(1 + \Delta)^3 e^{-2x\Delta} \right], \quad (32)$$

where $\sigma_{ij} = \sigma(L_i^0 L_j^0 \rightarrow \text{anything})$, and we have introduced

$$\Delta = \left| \frac{m_2}{m_1} \right| - 1; \quad (33a)$$

$$g_{\text{eff}} = 2 \left[1 + (1 + \Delta)^{3/2} e^{-x\Delta} \right]. \quad (33b)$$

g_{eff} is the effective number of degrees of freedom of CDM particles ($g = 2$ for a single Majorana fermion). Numerically, $x_f \simeq 20$ or so for models leading to an acceptable relic density.

Once x_f has been determined, the relic density is given by

$$\Omega h^2 = \frac{1.07 \cdot 10^9 \text{ GeV}^{-1}}{J(x_f) \sqrt{g_*} M_P}, \quad (34)$$

where the annihilation integral J is defined as

$$J(x_f) = \int_{x_f}^\infty \frac{\langle \sigma_{\text{eff}} v \rangle(x)}{x^2} dx. \quad (35)$$

As usual, we have expressed the relic density Ω in units of the critical (closure) density, so that $\Omega = 1$ corresponds to a flat universe as predicted by inflationary models [11]. Finally, h is the present Hubble parameter in units of 100 km/(s·Mpc); it lies in the range $0.4 \leq h \leq 1$. The constraint that the universe be older than 10 billion years implies $\Omega h^2 \leq 1$; the true upper bound is almost certainly tighter than this, but we want to be conservative since our calculation will only be precise on the 10% level.

In our calculation of the annihilation cross section σ_{ij} of eq.(32) we have only included annihilation into $f\bar{f}$ pairs, where f is a fermion contained in the Standard Model. This under-estimates the total annihilation cross section, and hence over-estimates the relic density, if L_1^0 is heavy enough to annihilate into pairs of gauge or Higgs bosons, or mixed gauge-Higgs final states. However, in this case annihilation into $f\bar{f}$ final states is by itself usually sufficient to give an acceptable relic density. The reason is that a large $|m_1|$ also implies fairly large $SU(2)$ doublet components of L_1^0 , and hence sizable couplings to gauge and Higgs bosons.

The process $L_i^0 L_j^0 \rightarrow f \bar{f}$ can proceed via the exchange of a neutral (scalar or pseudo-scalar) Higgs boson, or a Z boson, in the s -channel. As discussed in Sec. 3, there might also be terms in the superpotential that couple exotic leptons to (s)fermions present in the MSSM. However, the resulting contributions to the annihilation cross section are strongly suppressed. We already saw that bounds on rare processes severely constrain these couplings [19]. Moreover, they would contribute to annihilation only through the exchange of a sfermion in the t - or u -channel. The experimental lower bound on squark masses is already quite high [28]; furthermore, as mentioned earlier, one expects sfermion masses to be of order $M_{Z'}$ in this model [5, 6], which leads to a strong suppression of sfermion exchange contributions. Finally, these couplings involve the $SU(2)$ doublet components of L_i^0 , which are usually quite small for $i = 1, 2$; the corresponding contribution to the annihilation matrix element therefore involves two small mixing factors. This is also true for the Z -exchange contribution, but the $HL_i^0 L_j^0$ coupling (22) contains only one small mixing angle if the Higgs boson is mostly an $SU(2)$ doublet.[‡] We therefore neglect possible t -channel exchange diagrams.

Including the s -channel exchange of Z and Higgs bosons, the relevant matrix element can be written as

$$\mathcal{A}(L_i^0(k_1)L_j^0(k_2) \rightarrow f(p_1)\bar{f}(p_2)) = \mathcal{A}_{ij}^{(Z)} + \mathcal{A}_{ij}^{(A)} + \mathcal{A}_{ij}^{(h)}, \quad (36)$$

with

$$\mathcal{A}_{ij}^{(Z)} = i \left(\frac{g}{\cos\theta_W} \right)^2 t_{ij} \bar{v}(k_2) \gamma^\mu \gamma_5 u(k_1) \frac{g_{\mu\nu} - \frac{P_\mu P_\nu}{M_Z^2}}{s - M_Z^2 + i M_Z \Gamma_Z} \bar{u}(p_1) \gamma^\nu (a_f + b_f \gamma_5) v(p_2); \quad (37a)$$

$$\mathcal{A}_{ij}^{(A)} = i \frac{g}{\cos\theta_W} t_{ij}^{(A)} c^{(f)} \bar{v}(k_2) \gamma_5 u(k_1) \frac{1}{s - m_A^2 + i m_A \Gamma_A} \bar{u}(p_1) \gamma_5 v(p_2); \quad (37b)$$

$$\mathcal{A}_{ij}^{(H)} = i \frac{g}{\cos\theta_W} \bar{v}(k_2) u(k_1) \bar{u}(p_1) v(p_2) \sum_{k=1}^3 \frac{t_{ijk}^{(h)} d_k^{(f)}}{s - m_{h_k^0}^2 + i m_{h_k^0} \Gamma_{h_k^0}}. \quad (37c)$$

The $ZL_i^0 L_j^0$ couplings t_{ij} are given in eq.(7), while the $AL_i^0 L_j^0$ couplings $t_{ij}^{(A)}$ and the $h_k^0 L_i^0 L_j^0$ couplings $t_{ijk}^{(h)}$ can be read off eq.(22). The $Af\bar{f}$ couplings $c^{(f)}$ and $h_k^0 f\bar{f}$ couplings $d_k^{(f)}$ are listed in eqs.(19a,19b). Finally, the $Zf\bar{f}$ couplings a_f and b_f are as usual given by $a_f = -\frac{1}{2}I_3^f + q^f \sin^2 \theta_W$, $b_f = \frac{1}{2}I_3^f$, with I_3^f and q^f the weak isospin and electric charge of fermion f , respectively. In eq.(37a) we have introduced the total momentum $P_\mu = (k_1 + k_2)_\mu = (p_1 + p_2)_\mu$, and $s = P^\mu P_\mu$.

The A and Z exchange diagrams interfere with each other, but not with the scalar Higgs exchange contribution. The annihilation cross section can most readily be computed using standard trace techniques. The result can be written as

$$\sigma(L_i^0 L_j^0 \rightarrow f \bar{f}) v = \frac{\beta_f N_f}{8\pi s} \left[\tilde{\sigma}_{ij}^{(ZZ)} + \tilde{\sigma}_{ij}^{(ZA)} + \tilde{\sigma}_{ij}^{(AA)} + \tilde{\sigma}_{ij}^{(hh)} \right], \quad (38)$$

[‡]Recall that this coupling results from the superpotential (1), which couples one singlet to two doublets. An $SU(2)$ doublet Higgs boson therefore couples to $L_i^0 L_j^0$ via one singlet and one doublet component. Recall also that in this model all light Higgs bosons must be dominantly doublets, since $x \gg v, \bar{v}$ in eq.(9), so that the singlet Higgs boson essentially decouples from physics at scale M_Z .

where $\beta_f = \sqrt{1 - 4m_f^2/s}$, $N_f = 1$ (3) for leptons (quarks), and the scaled annihilation cross sections $\tilde{\sigma}_{ij}$ are given by:

$$\tilde{\sigma}_{ij}^{(ZZ)} = 4 \left(\frac{g}{\cos\theta_W} \right)^4 (t_{ij})^2 (a_f^2 + b_f^2) \left[\frac{s^2/4 + s\beta_f^2 \vec{k}^2/3 - s(m_f^2 + m_i m_j) + 4m_f^2 m_i m_j}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \frac{2m_f^2}{M_Z^2} \left(1 - \frac{s}{2M_Z^2} \right) \frac{(m_i^2 - m_j^2)^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \frac{m_f^2 (m_i + m_j)^2}{M_Z^4} \right]; \quad (39a)$$

$$\tilde{\sigma}_{ij}^{(ZA)} = 4 \left(\frac{g}{\cos\theta_W} \right)^2 t_{ij} t_{ij}^{(A)} c^{(f)} b_f m_f (m_i + m_j) \frac{s - m_A^2}{(s - m_A^2)^2 + m_A^2 \Gamma_A^2} \frac{s - (m_i - m_j)^2}{M_Z^2}; \quad (39b)$$

$$\tilde{\sigma}_{ij}^{(AA)} = \left(\frac{g}{\cos\theta_W} \right)^2 (t_{ij}^{(A)} c^{(f)})^2 \frac{s - (m_i - m_j)^2}{(s - m_A^2)^2 + m_A^2 \Gamma_A^2}; \quad (39c)$$

$$\tilde{\sigma}_{ij}^{(hh)} = \left(\frac{g}{\cos\theta_W} \right)^2 [s - (m_i + m_j)^2] s \beta_f^2 \left| \sum_{k=1}^3 \frac{t_{ijk}^{(h)} d_k^{(f)}}{s - m_{h_k^0}^2 + i m_{h_k^0} \Gamma_{h_k^0}} \right|^2. \quad (39d)$$

Here, the initial 3-momentum \vec{k}^2 is again given by eq.(24) with M_Z^2 replaced by s . As before, the masses m_i and m_j can have either sign.

A few comments are in order. When writing the thermal average (31), we have used non-relativistic kinematics; for consistency we therefore also have to use a non-relativistic expression for s in eqs.(39),

$$s = (|m_i| + |m_j|)^2 + |m_i m_j| v^2. \quad (40)$$

this might seem dangerous, since in the presence of narrow poles the integral in eq.(31) can receive sizable contributions from $v \sim 1$. However, we have checked that using fully relativistic kinematics everywhere does not change the result significantly; on the other hand, combining eq.(31) with a relativistic expression for s can under-estimate the relic density by a factor of 2 or 3.

In principle we now could compute Ωh^2 numerically, by inserting eqs.(38) and (39) into eqs.(30)–(35). Note, however, that inserting eq.(31) into eq.(35) leads to a double integration. Since we want to test several million combinations of model parameters, in order to make sure that we covered all relevant regions of parameter space, a direct numerical integration is not practical. We used the following approximate method instead.

As well known [11, 27], the integral in eqs.(31) and (35) can be computed quite reliably from a simple analytical expression *if* the annihilation cross section σ_{eff} does not depend too sensitively on v . In this case one can use the Taylor expansion

$$\sigma_{\text{eff}} = a_{\text{eff}} + b_{\text{eff}} v^2, \quad (41)$$

which gives $\langle \sigma_{\text{eff}} v \rangle(x) = a_{\text{eff}} + 6b_{\text{eff}}/x$, and $J(x_f) = a_{\text{eff}}/x_f + 3b_{\text{eff}}/x_f^2$. Since $x_f \simeq 20$, annihilation from an s -wave initial state, which contributes to a_{eff} , reduces the relic density more efficiently than annihilation from a p -wave does, which only contributes to b_{eff} . However, in our case this expansion can be used with some reliability only for those contributions that do not have an s -channel pole. Specifically, this includes the last term in the squared Z

exchange contribution (39a), which is due to the longitudinal polarization state of the Z ; the $Z - A$ interference term (39b); and the $H_k^0 - H_l^0$ interference terms in eq.(39d). All other contributions do in general show strong variation with v^2 , and have to be treated separately.

Since we use the expansion (41) for all interference terms, we now only need to compute the thermal average over a Breit–Wigner propagator, multiplied with a power of v^2 :

$$I_n \equiv C \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv e^{-xv^2/4} \frac{(v^2)^{1+n} s(v)}{[s(v) - m_P^2]^2 + m_P^2 \Gamma_P^2}, \quad (42)$$

where $s(v)$ is given by eq.(40), C is a constant, and $P = Z, A$ or h_k^0 . Setting for simplicity $s = (|m_i| + |m_j|)^2$ in the numerator (but not the denominator) of eq.(42), I_n can be written as

$$I_n = C \frac{x^{3/2}}{2\sqrt{\pi}} \left(\frac{|m_i| + |m_j|}{m_i m_j} \right)^2 \int_0^\infty dv e^{-xv^2/4} \frac{(v^2)^{1+n}}{(v^2 - v_0^2)^2 + \gamma}, \quad (43)$$

with

$$v_0^2 = \frac{m_P^2 - (|m_i| + |m_j|)^2}{|m_i m_j|}; \quad (44a)$$

$$\gamma = \left(\frac{m_P \Gamma_P}{m_i m_j} \right)^2. \quad (44b)$$

Note that $v_0^2 < 0$ implies $s > m_P^2$ for all $v \geq 0$, so that the pole is never accessible. On the other hand, if $v_0^2 > 0$, the contribution from $v \simeq v_0$ to the integral in eq.(43) scales like $e^{-xv_0^2/4} (v_0^2)^{1+n} \gamma^{-0.5}$, which can be substantial even for $v_0 \sim 1$ if $\gamma \ll 1$, i.e. if the pole is very narrow.

The integral in eq.(43) still seems to depend on three parameters (the inverse temperature, and the position and width of the pole). Further progress can be made by using the substitution $v' = \sqrt{x}v$:

$$\begin{aligned} I_n &= C \frac{x^{2-n}}{2\sqrt{\pi}} \left(\frac{|m_i| + |m_j|}{m_i m_j} \right)^2 \int_0^\infty dv' e^{-v'^2/4} \frac{(v'^2)^{1+n}}{(v'^2 - \tilde{v}_0^2)^2 + \tilde{\gamma}} \\ &\equiv C x^{2-n} \left(\frac{|m_i| + |m_j|}{m_i m_j} \right)^2 \tilde{I}_n(\tilde{v}_0^2, \tilde{\gamma}), \end{aligned} \quad (45)$$

with $\tilde{v}_0^2 = xv_0^2$ and $\tilde{\gamma} = x^2\gamma$. We have computed \tilde{I}_0 and \tilde{I}_1 numerically for 200 values of \tilde{v}_0^2 between -100 and $+225$, and 50 values of $\tilde{\gamma}$ between $6.3 \cdot 10^{-3}$ and $3.5 \cdot 10^3$.[§] Note that these 10,000 values of \tilde{I}_0 and \tilde{I}_1 need to be computed only once; afterwards the thermal average over the annihilation cross section can be computed without numerical integration: If \tilde{v}_0^2 does not lie in this range, the pole is so distant that the expansion (41) can be used for it; for values of \tilde{v}_0^2 and $\tilde{\gamma}$ in the specified range, the \tilde{I}_n are estimated by interpolation.

Specifically, for $i = j$ ($L_1^0 L_1^0$ or $L_2^0 L_2^0$ annihilation), only the squared A exchange term (39c) gets contributions $\propto \tilde{I}_0$ when inserted in eq.(31); the resonant contributions to eq.(39a),

[§]Recall that we need to compute I_n only for $x \geq 20$. Moreover, $\Gamma_P/m_P \geq 10^{-4}$ even for the light Higgs bosons of the model.

as well as eq.(39d), are proportional to v^2 for $m_i = m_j$, i.e. only contribute via \tilde{I}_1 . The reason is that on-shell Z and scalar Higgs bosons can only be produced from two identical Majorana fermions if they are in a p -wave state, while on-shell pseudo-scalar bosons can be produced from an s -wave state. However, squared Z and scalar Higgs exchange do contribute to \tilde{I}_0 terms if $m_i \neq m_j$. In the important special case where L_1^0 and L_2^0 form a Dirac fermion, one has $m_1 = -m_2$; in this case squared A exchange only starts at order v^2 (\tilde{I}_1 terms only), while squared scalar Higgs and Z exchange start at order v^0 . Finally, for all contributions that start at order v^0 (\tilde{I}_0 terms), we have expanded the numerator to order v^2 , i.e. added a (properly normalized) \tilde{I}_1 term. We checked numerically that our combination of eq.(41) for non-resonant (interference) terms, and eq.(45) with interpolation to the actual values of \tilde{v}_0^2 and $\tilde{\gamma}$, reproduces the exact thermal average of the annihilation cross section to an accuracy of about 10% or better; this is quite sufficient for us. In contrast, simply using the expansion (41) for the entire cross section can both over- and under-estimate the true thermal average by a large factor; this had been observed previously [29] in the similar case of the MSSM with scalar Higgs mass close to twice the LSP mass. Finally, the annihilation integral (35) usually converges rather quickly; we have therefore computed it numerically, still using the method outlined above to determine $\langle\sigma_{\text{eff}}v\rangle(x)$ in the integrand.

5) Results

We are now in a position to describe our numerical studies of the model, using the expressions given in the previous three sections. Our basic approach is to randomly sample the parameter space, and to count the number of solutions that satisfy all the constraints we impose, including those that can be derived from searches at future (linear) e^+e^- colliders. The ultimate goal is to devise a set of constraints such that there are no acceptable solutions left. This means that either a signal characteristic for the model has been found, i.e. one of the hypothetical future searches is successful, or the model is completely excluded, independent of the values of the free parameters.

We chose this Monte Carlo approach since the number of free parameters is too large to allow for a systematic scan of the entire parameter space. The mass matrix (3) for the neutral exotic leptons already contains ten free parameters. For each scan of parameter space, we impose a lower bound on the charged leptons masses $m_{L_i^\pm}$, since these particles should be trivial to discover at an e^+e^- collider unless their production is kinematically suppressed. We also fix the ratio $\tan\beta \equiv v/\bar{v}$ for a given run; this determines v and \bar{v} , since $\sqrt{v^2 + \bar{v}^2} = 2M_W/g$ is known. The Yukawa couplings appearing in eq.(3) are then chosen randomly in the interval $|\lambda_{ijk}| \leq \lambda_{\text{max}}$. We take $\lambda_{\text{max}} = 0.85$, which is approximately the upper bound on any one coupling from the requirement that there should be no Landau pole at scales below the GUT scale. This is a conservative approach, since this bound is significantly stronger if several Yukawa couplings are sizable [10], which is required if $m_{L_1^0}$ is to be close to its upper bound.

Having specified the exotic lepton sector, we check whether the present LEP constraints (25) and (26) are satisfied. If so, we check whether the total “visible” exotic cross-section, determined from eq.(27), is sufficiently small so that a given e^+e^- collider will not detect pair production of exotic leptons; this constraint obviously depends on the collider we are

considering, as described below. Recall that we consider a neutral exotic lepton to be “visible” only if it is at least 5 GeV heavier than L_1^0 .

This exhausts the constraints from searches for exotic leptons at present and future e^+e^- colliders. If our choice of parameters is still viable, we next randomly pick a Higgs sector, subject to the constraints (28). Since present bounds on the mass of the Z' boson are already quite high [2], we always find that there is one singlet-like neutral Higgs boson, which plays no role in any of the process we are considering; it is therefore sufficient to simply fix the SM singlet vev x to some large value, say 2 TeV. Since our runs are for fixed $\tan\beta$, we only need to choose the values of two additional parameters in order to completely specify the Higgs sector of the model, see eq.(8). We chose these to be the coupling λ_{333} , which also has to lie in the interval $|\lambda_{333}| \leq 0.85$, and the mass m_A of the physical pseudo-scalar Higgs boson; this then fixes the parameter A_{333} via eq.(12). Note that for each set of leptonic parameters that satisfy constraints from e^+e^- colliders, we keep choosing pairs (λ_{333}, m_A) at random until we have found an acceptable Higgs sector.

This is necessary because Higgs exchange contributions can play an important role in the calculation of the L_1^0 relic density. Once the exotic lepton sector and the Higgs sector are specified, all quantities appearing in the annihilation cross sections (39) are fixed, and we can compute the relic density as described in Sec. 4, and check whether it is acceptable.

Figs. 1a,b show that present constraints [the bounds (25), (26) and (28), together with $\Omega h^2 \leq 1$] are still quite far from testing the model decisively. For these figures we have allowed charged lepton masses as low as 40 GeV when sampling the parameter space; we have however required $|m_{L_i^\pm}| \geq 45$ GeV for all acceptable solutions. This explains why the survival fraction after imposing LEP1 constraints only is less than unity in some bins with $m_{L_1^0} > M_Z/2$.

Note that LEP1 searches only impose weak constraints for very small values of $m_{L_1^0}$. The reason is that most parameter sets with small $m_{L_1^0}$ have large masses for the charged exotic leptons. In this case four of the six neutral exotics are also heavy, while the remaining two states are mostly SM singlets, i.e. couple only weakly to the Z boson. However, the relic density constraint is most effective precisely in this situation, since it leads to small annihilation cross sections: The couplings of singlet-like exotics to the light Higgs bosons are also weak, and their small masses suppress the cross sections even further; far below the pole, s -channel exchange contributions scale like $m_{L_i^0}^2/m_P^4$, where P is the exchanged particle. We therefore find no cosmologically acceptable scenario with $m_{L_1^0} \leq m_b \simeq 5$ GeV; this is not surprising since the $b\bar{b}$ final state, if accessible, contributes most to Higgs exchange diagrams.

Figs. 1 also reveal a technical problem: even though each figure is based on 10^5 sets of leptonic parameters (“hundred thousand models”), the region of large $m_{L_1^0}$ is only sparsely populated. This is perhaps not surprising, since all ten parameters appearing in the mass matrix (3) must be chosen within a narrow range if $m_{L_1^0}$ is to come out close to its upper bound. At the same time, combinations of parameters leading to large $m_{L_1^0}$ are most difficult to exclude, i.e. most easily evade all bounds. For one thing, large $m_{L_1^0}$ implies large $SU(2)$ doublet components of L_1^0 , and hence large annihilation cross sections and a small relic density. Further, constraints from collider searches can most easily be evaded if L_2^0 is close in mass to L_1^0 , since L_2^0 then also becomes effectively invisible. In such a situation $L_1^0 L_2^0$ co-annihilation can also reduce the relic density even further. Since our goal is to test the

model decisively, it would be advantageous if in our sampling of parameter space we could give preference to regions that are most difficult to exclude.

Fortunately a large $m_{L_1^0}$ is correlated with a small $L_1^0 - L_2^0$ mass difference. In ref.[10] it has been shown that choices of parameters that maximize $m_{L_1^0}$ always lead to a situation where the six eigenvalues of the mass matrix (3) come in three pairs that only differ by a sign; in such a situation the six Majorana states can also be described by three neutral Dirac fermions. In particular, the Majorana state L_2^0 will now be completely invisible, being degenerate in mass with L_1^0 and hence (by assumption) stable; this obviously also maximizes $L_1^0 L_2^0$ co-annihilation. From now on we will therefore only show results for the subset of parameter space where the six neutral exotic leptons do indeed form three Dirac fermions. This can be enforced by choosing

$$\lambda_{131} = \lambda_{132}, \lambda_{231} = \lambda_{232}, \lambda_{311} = -\lambda_{312}, \lambda_{321} = -\lambda_{322}. \quad (46)$$

For given $\tan\beta$, this reduces the number of parameters in the exotic lepton sector from ten to six; this reduced parameter space is obviously much easier to sample exhaustively. We have checked that our runs with eqs.(46) imposed do always find significantly more acceptable solutions than scans of the entire parameter space with equal statistics.

Figs. 2a,b show that, at least if the mass matrix (3) has Dirac structure, neutral lepton searches at LEP2 will not lead to significant new constraints, unless a light charged exotic is found. We have assumed here that LEP2 will reach a center-of-mass energy \sqrt{s} of 190 GeV, and that the production of neutral leptons is detectable if the cross section, summed over all “visible” modes, exceeds 20 fb, which corresponds to 10 events per experiment for the foreseen integrated luminosity of 500 pb⁻¹. Comparison with Figs. 1 shows that the fraction of parameter space excluded by present LEP1 constraints has become smaller, even for light L_1^0 . This is mostly because $L_1^0 L_2^0$ and $L_2^0 L_2^0$ final states are now invisible. Note also that we now find some cosmologically acceptable solutions with $\tan\beta = 1.2$ and $m_{L_1^0}$ as small as 10 GeV; this indicates that co-annihilation can indeed reduce the relic density significantly. Further, even though the absolute upper bound on $m_{L_1^0}$ decreases with increasing mass of the charged exotic leptons [10], Figs. 2a,b extend to larger values of $m_{L_1^0}$ than Figs 1a,b do; clearly the restrictions (46) have made it much more likely to produce scenarios with large $m_{L_1^0}$. Finally, comparison of Figs. 2a and 2b shows that LEP constraints exclude a larger fraction of parameter space for large $\tan\beta$; this can already be seen from Figs. 1a,b. We showed in Sec. 2 that increasing $\tan\beta$ decreases the upper bound on $m_{L_1^0}$, since $\det \mathcal{M}_{L^0} \propto (v\bar{v}x)^2$; however, the size of the $SU(2)$ doublet components of the light exotics, and hence their couplings to the Z boson, remains more or less the same. Reducing the masses of the exotics while leaving their couplings essentially unchanged obviously increases the partial width for Z decays into exotic leptons.

Figs. 2 clearly show that LEP2 will not be able to test the model decisively; there are many choices of parameters that lead to an acceptable cosmology, but no “new physics” signal at this collider. One will therefore need (linear) colliders operating at higher energies in order to probe the entire parameter space. Such colliders are often assumed to be built in three stages, where the energy is increased from about 0.5 TeV to 1.0 TeV and, eventually, 1.5 TeV or even higher. We generically call these three stages NLC1, NLC2 and NLC3.

Figs. 3a,b show the situation if a 500 GeV e^+e^- collider fails to discover pair production of exotic leptons, which we have interpreted as meaning that the total cross section into

visible exotic final states is less than 0.5 fb; the integrated luminosity of such a collider is usually assumed to be several tens of fb^{-1} per year. For these runs we have required the masses of the charged exotic leptons to exceed 240 GeV. This reduces both the upper bound on $m_{L_1^0}$ and the maximal $SU(2)$ doublet component of L_1^0 significantly. As a result, for $\tan\beta = 1.2$ present LEP1 searches do not constrain the model any further. For $\tan\beta = 5$, these constraints still do exclude some combinations of parameters, but they are clearly much less restrictive here than for lower masses of the charged exotics, see Fig. 2b. On the other hand, unlike at LEP2, searches for neutral exotics at NLC1 can probe sizable regions of parameter space even if no charged exotic leptons are found; this can be seen from the large differences between the dotted and dashed histograms in Figs. 3. Finally, the relic density constraint again excludes parameter choices giving $m_{L_1^0} \leq m_b$, and impose significant constraints as long as $m_{L_1^0} \leq 15$ to 20 GeV. However, a substantial number of parameter sets still satisfies all constraints (solid histograms).

This remains true even if a 1 TeV collider fails to discover pair production of exotic leptons. Indeed, Figs. 4a,b show that a small region of parameter space survives even if searches for pairs of exotic leptons at a 1.5 TeV collider remain unsuccessful; here we have assumed that the integrated luminosity scales like the square of the beam energy, so that a signal exceeding 0.05 fb would be detectable. We evidently need to find some additional constraint(s) if we want to test the model decisively.

There are significant differences between Figs. 4a,b and 3a,b, which might offer a clue as to what these additional constraints might be. To begin with, the minimal allowed value of $m_{L_1^0}$ for small $\tan\beta$ has increased from about 6 GeV (Fig. 3a) to about 23 GeV (Fig. 4a). Unsuccessful searches at a 1.5 TeV collider force the $SU(2)$ doublet components of L_1^0 to be much smaller than searches at an 0.5 TeV collider do, which leads to considerably reduced couplings of the light exotic leptons to gauge and Higgs bosons. This has to be compensated by an increase of $m_{L_1^0}$ in order to keep the relic density acceptably small; recall that for light L_1^0 , the annihilation cross sections scale like $m_{L_1^0}^2$. Even more importantly, if a 1.5 TeV collider fails to find evidence for the pair production of exotic leptons, the upper bound on $m_{L_1^0}$ is reduced to about 30 (11) GeV for $\tan\beta = 1.2$ (5), as compared to 84 (27) GeV after unsuccessful searches at NLC1.

Recall that the masses of the two lightest exotic Majorana states, or the lightest exotic Dirac state if eqs.(46) hold, come from vevs that break $SU(2)$. This indicates that there is a correlation between $m_{L_1^0}$ and the size of the couplings between light exotics and $SU(2)$ doublet Higgs bosons, although the relation is not as simple as that between the masses and Yukawa couplings of the quarks and leptons of the SM. Further, Higgs searches at LEP1 imply that the mass of a scalar Higgs boson with unsuppressed ZZH coupling must exceed 60 GeV, see eq.(28a). Such a Higgs boson can therefore decay into pairs of light neutral exotic leptons for all allowed combinations of parameters shown in Figs. 4; this could lead to a large invisible branching ratio of this Higgs boson. Indeed, we find that all surviving scenarios shown in Fig. 4a have one rather light neutral scalar Higgs boson, with essentially unsuppressed coupling to two Z bosons, and with invisible branching ratio exceeding 80%. Moreover, it will be produced copiously at any e^+e^- collider with $\sqrt{s} \geq 300$ GeV, since its mass cannot exceed 150 GeV or so, see eq.(10). Note that such a Higgs boson is easily detectable even if it decays invisibly [30], since it would be produced in association with a Z boson, whose decay products would be sufficient to reconstruct m_H . Finally, a large invisible

branching ratio for a light scalar Higgs boson cannot be accommodated in the SM; in the MSSM it would imply the existence of a chargino light enough to be discovered at LEP2 [24]. It therefore constitutes a signal for the kind of model we are considering.

The situation at large $\tan\beta$, Fig. 4b, is somewhat more complicated. Notice that, unlike in Fig. 4a, now only a small fraction of all parameter sets that give $m_{L_1^0}$ close to its upper bound, and do not lead to a detectable signal for exotic lepton pair production, survives the relic density constraint. The reason is that for small $\tan\beta$ and with $m_{L_1^0}$ close to its upper bound, Z exchange by itself is usually sufficient to give an acceptable relic density. On the other hand, increasing $\tan\beta$ to 5 reduces the maximal $m_{L_1^0}$ so much that Z exchange, the cross section for which scales like $m_{L_1^0}^2/M_Z^4$, is no longer sufficient; light Higgs bosons have to be present to enhance the annihilation cross section. In particular, a light pseudo-scalar Higgs boson A allows $L_1^0 L_1^0$ and $L_2^0 L_2^0$ to annihilate from an s -wave initial state, which gives a larger thermal average than annihilation from a p -wave initial state does, as discussed below eq.(41) in Sec. 4.* Therefore even if $m_{L_1^0}$ is close to its upper bound, the relic density will only be acceptable if m_A is chosen to be fairly small, which is true only for some fraction of Higgs parameter space. We find that all surviving parameter sets in Fig. 4b have $m_A < 110$ GeV, and also have two neutral scalar Higgs bosons with masses below 125 GeV. At least one of these three light neutral Higgs bosons has invisible branching ratio exceeding 50%. All three of these Higgs bosons will be produced copiously, either in association with a Z boson or as Ah_i^0 pairs. A large invisible branching ratio for any of these Higgs particles is therefore again a distinctive signature. Note that now the invisible branching ratio is always less than 75%, so that in at least 50% of all Ah_i^0 pairs at least one of the two Higgs bosons will decay into a visible final state, mostly $b\bar{b}$ pairs. This not only guarantees the detectability of this final state, but also ensures that the invisible branching ratios can be measured with some accuracy, e.g. by comparing the rate for events with one invisible Higgs boson (single-sided events) with that for events where both Higgs particles leave a detectable final state. Finally, we note that in the surviving cases the charged Higgs boson mass (13) also is below 125 GeV, and usually below 100 GeV; such a light charged Higgs boson is already almost excluded in the MSSM, and might thus give a second indication for physics beyond the MSSM.

Based on the results of Figs. 4 we therefore conclude that in this model either a signal for the pair production of exotic leptons will be observed, at the latest at an e^+e^- collider with $\sqrt{s} = 1.5$ TeV; or at least one neutral Higgs boson with mass below 150 GeV and invisible branching ratio exceeding 50% will be found at any e^+e^- collider with $\sqrt{s} \geq 300$ GeV. If neither of these two signals is detected, the model can be completely excluded.

Since this conclusion is based on a large but finite Monte Carlo sampling of parameter space (Figs. 4a,b contain $5 \cdot 10^5$ sets of leptonic parameters each), we have checked that it also holds in a “worst case” scenario. To this end we have chosen the parameters entering the mass matrix (3) such that $m_{L_1^0}$ is maximized [10]:

$$\lambda_{131} = \lambda_{132} = \lambda_{311} = -\lambda_{312} = \lambda_{231} = -\lambda_{321} = \lambda_{232} = \lambda_{322} = \lambda_{\max}; \quad (47a)$$

$$m_{L_1^\pm} = -m_{L_2^\pm}. \quad (47b)$$

* $L_1^0 L_2^0$ co-annihilation via scalar Higgs exchange could come from an s -wave initial state; however, if the neutral lepton mass matrix has Dirac structure, the off-diagonal $L_1^0 L_2^0 h_i^0$ couplings vanish identically, as does the $L_1^0 L_2^0 A$ coupling.

Note that this ansatz is consistent with eqs.(46). In Fig. 5 we show the resulting maximal value of $m_{L_1^0}$ as a function of $\tan\beta$, subject to the constraints $m_{L_1^\pm} > m_{L^\pm, \min}$ and $|m_{L_1^0}| + |m_{L_3^0}| > 2m_{L^\pm, \min}$. The latter bound approximates the constraint that can be derived from an unsuccessful search for associate $L_1^0 L_3^0$ production; note that the relevant coupling is always quite large for the ansatz (47).[†] For our choice $\lambda_{\max} = 0.85$, the maximal allowed $m_{L_1^0}$ for $m_{L^\pm, \min} = 700$ GeV is indeed around 30 GeV, in agreement with results shown in Fig. 4a; for $\tan\beta = 5$, $m_{L_1^0, \max}$ falls to about 11 GeV, in agreement with Fig. 4b. We should mention here that the choice $\lambda_{\max} = 0.85$ in eq.(47a) is very conservative. In ref.[10] it has been pointed out that $\lambda_{\max} > 0.7$ implies the existence of a Landau pole at a scale below 10^{10} GeV, the smallest energy scale where one might expect the gauge group (and hence the relevant renormalization group equations) to change in this class of models. Note that for large $m_{L^\pm, \min}$, the upper bound on $m_{L_1^0}$ scales like the square of λ_{\max} ; reducing λ_{\max} to 0.7 therefore means that $m_{L_1^0} \leq 30$ GeV already for $m_{L^\pm, \min} = 480$ GeV, in which case a 1 TeV collider would be sufficient to test the model decisively in the manner described above.[‡]

We also checked whether it is possible to chose parameters such that the SM-like Higgs boson only has very weak couplings to light neutral exotics. This is indeed possible, but only for sizable $\tan\beta$, and only if the pseudoscalar Higgs boson is quite light. In this case the model contains two light $SU(2)$ doublet neutral scalar Higgs bosons, as mentioned earlier; the “SM-like Higgs” is defined to be the one with the larger coupling to two Z bosons. We saw in Fig. 4b that for large $\tan\beta$, the relic density constraint can only be satisfied if m_A is rather small; the conditions for a scenario with small invisible width of the SM-like Higgs are therefore satisfied. However, Fig. 6 shows that either the pseudoscalar Higgs or the lightest neutral scalar can always decay into an $L_1^0 L_1^0$ pair if searches at a 1.5 TeV collider do not find a signal for the pair production of exotic leptons. In this figure we show the minimal allowed values of m_A and of $m_{h_1^0}$, as well as the minimum of the sum of these two masses, as a function of $\tan\beta$, where we have only used the present LEP1 constraints (28). For $\tan\beta \simeq 1.5$, m_A could be as low as 34 GeV, so that $A \rightarrow L_1^0 L_1^0$ decays would be kinematically forbidden over a sizable region of parameter space. However, in this case the lightest scalar Higgs boson must be heavier than 50 GeV, since $m_A + m_{h_1^0} > 84$ GeV. Note that in this scenario, h_1^0 is usually not the SM-like Higgs, and does have sizable couplings to light exotic leptons. Moreover, we find that the invisible branching ratio of the SM-like scalar Higgs can only be reduced to a value below 10% if $\tan\beta \geq 3$, in which case unsuccessful searches for lepton pair production at NLC3 imply $m_{L_1^0} < 20$ GeV, see Fig. 5. In this case both A and h_1^0 have large invisible branching ratios; remember that a sizable contribution from A exchange to L_1^0 annihilation, and hence a substantial $AL_1^0 L_1^0$ coupling, is required to satisfy the relic density constraint for $\tan\beta > 2$. Finally, we found no acceptable solutions where the invisible branching ratio of a light Higgs boson can be diluted by decays into pairs of even lighter Higgs bosons ($h_2^0 \rightarrow h_1^0 h_1^0$ or $h_2^0 \rightarrow AA$). We conclude that it is indeed impossible to devise a Higgs sector such that no light Higgs boson has large invisible branching ratio if NLC3 fails to find a signal for the pair production of exotic leptons.

Finally, we investigated the question whether Higgs searches at LEP2 will allow us to

[†]Recall that the production of two light neutral exotic leptons leads to an invisible final state.

[‡]The larger value of λ_{\max} we are using here also explains why Fig. 5 allows larger values of $m_{L_1^0}$ than ref.[10] does.

sharpen our predictions. This does not seem to be the case, at least as far as searches at a 1.5 TeV collider are concerned. As mentioned earlier, for small $\tan\beta$, Z exchange by itself can be sufficient to give an acceptable relic density, so increasing the experimental lower bounds on Higgs masses will have little effect on Fig. 4a. For $\tan\beta \geq 3$ one needs $m_A \leq 110$ GeV in order to achieve $\Omega h^2 \leq 1$. This upper bound is almost independent of $\tan\beta$ once it exceeds 3 or so; the decrease of $m_{L_1^0}$ caused by increasing $\tan\beta$ is balanced almost perfectly by the increase of the $A b \bar{b}$ and $A \tau^+ \tau^-$ couplings, leading to a constant A exchange contribution, as long as $m_{L_1^0} > m_b$. However, this bound still allows values of m_A just beyond the reach of LEP2.

On the other hand, if LEP2 can increase the lower bound on the SM Higgs to 90 GeV or more, a 1 TeV e^+e^- collider would be sufficient to test the model decisively even with our choice $\lambda_{\max} = 0.85$, since the absence of a signal for exotic lepton pair production at such a collider implies $m_{L_1^0} < 45$ GeV, see Fig. 5; one would then again be in the situation where at least one copiously produced Higgs boson has to have a large invisible branching ratio. Unfortunately in the absence of a positive signal for lepton pair production, a 0.5 TeV collider would never be sufficient to test the model uniquely, since, as shown in Fig. 3a, it would only allow to establish the bound $m_{L_1^0} < 85$ GeV, well above half the maximal allowed mass of the SM-like Higgs boson. One can then always find cosmologically acceptable scenarios where only one Higgs boson is experimentally accessible, with small or vanishing invisible branching ratio; such a model would be indistinguishable from the MSSM, or even the SM, as far as the NLC1 is concerned.

6) Summary and Conclusions

In this paper we have shown that searches for exotic leptons and Higgs bosons at future e^+e^- colliders, when combined with cosmological constraints, can test a large class of $E(6)$ models decisively. The basic idea is that, due to the structure of the mass matrix for the 6 neutral exotic leptons predicted by the model, the failure to detect the production of heavy (charged or neutral) exotic leptons at such colliders will allow to derive increasingly stronger upper bounds on the masses of the light neutral exotics. Eventually these leptons will have to be lighter than half the mass of the light neutral Higgs bosons of the model. The only way to suppress the couplings of these light exotic leptons to all light Higgs bosons is to make the leptons almost pure $SU(2)$ singlets. However, in this case exotic leptons produced in the very early universe would have small annihilation cross sections, so that their present relic density would be unacceptably large. All cosmologically acceptable parameter sets therefore predict that either the pair production of exotic leptons is visible at a 1.5 TeV e^+e^- collider, or at least one light neutral Higgs boson will have an invisible branching ratio exceeding 50%; if LEP2 fails to discover a Higgs boson, a 1 TeV e^+e^- collider would be sufficient.

In order to arrive at this conclusion, we had to make two crucial assumptions. First, no $SO(10)$ singlet scalar N is allowed to have a vev at an “intermediate scale” of $\mathcal{O}(10^{10})$ GeV or more. This assumption is not unreasonable, since such a large vev would allow to push the masses of almost all new particles predicted by $E(6)$ up to this high scale, including the new gauge bosons, the exotic quarks and leptons, and their superpartners. Apart from the possible presence of light right-handed (s)neutrinos, the model would then look like the

MSSM at scales below this vev. Note that we do allow $SO(10)$ nonsinglets to have such a large vev. Secondly, we have to assume that R -parity is conserved. Otherwise the lightest exotic lepton, which has odd R -parity, could decay even if it is lighter than the lightest neutralino, in which case the relic density constraint would be satisfied trivially. However, if R -parity is broken, this rather complicated model would not be able to accommodate cold dark matter; recent attempts to understand structure formation in the universe strongly favor scenarios with a substantial amount of cold dark matter [26].

We also made a few additional assumptions in order to simplify our calculation. However, since we have been very conservative in our interpretation of the cosmological constraint, and of the upper bound on the Yukawa couplings of the exotic leptons that follows from the requirement that no coupling should have a Landau pole below the intermediate scale of 10^{10} GeV, we believe that our final result holds even if these additional, technical assumptions are relaxed. Specifically, we have assumed that the exotic leptons do not mix with charginos and neutralinos. This assumption is technically natural. Moreover, in ref.[10] it has been argued that, while such mixing might increase the upper bound on $m_{L_1^0}$ for *fixed* Yukawa couplings, it would also necessitate the existence of additional terms in the superpotential; these would lower the upper bound on the relevant Yukawa couplings from the absence of Landau poles. The total change of the upper bound on $m_{L_1^0}$ is therefore quite modest.

Further, when computing the annihilation cross sections, we have assumed that there are no terms in the superpotential which couple exotic leptons to ordinary quark and lepton superfields. However, we argued in Sec. 4 that such couplings would in any case contribute negligibly to the total annihilation cross section. Finally, we have taken the Z' boson(s) of the model to be too heavy to contribute either to the pair production or to the annihilation of exotic leptons. Present lower bounds on $M_{Z'}$ already ensure that Z' exchange contributions to the annihilation cross sections are negligible. They could still affect the production of exotic leptons at TeV-scale e^+e^- colliders, but in this case the existence of a Z' boson could also be inferred from studies of ordinary quark and lepton pair production, which would again give a good signal for the class of models we are studying. Besides, the constraints on the exotic lepton sector that could be derived from new particle searches at future colliders are likely to be more, not less, severe if there is a significant contribution from Z' exchange, since even the $SU(2)$ singlet fields \widetilde{N} couple to the Z' boson with full gauge strength.

There is one more assumption that we have not mentioned so far: when using the formalism described in Sec. 4 to estimate the relic density, we have assumed that the exotic leptons are non-relativistic (“cold”) when they drop out of thermal equilibrium. This is true if their masses exceed a few hundred MeV or so, and the relic density constraint will remain valid down to much smaller masses, in the keV range, but our calculation clearly breaks down if the light exotics are (nearly) massless. This could, for example, be achieved by setting all couplings in the superpotential (1) to zero, except for λ_{113} and λ_{223} which are needed to give masses to the charged exotic leptons. The two lightest neutral exotics would then be massless $SU(2) \times U(1)_Y$ singlets. However, this would increase the density of relativistic particles, and hence the expansion rate of the universe, in the epoch when light nuclei are formed. The most recent analysis [31] finds that data seem to favor models where the effective number of SM neutrinos is smaller, not larger, than three. A model with additional massless fermions is therefore strongly disfavored.

Finally, it can be argued that the upper bound (10) on the mass of the lightest neutral

scalar Higgs boson of the model allows a much easier test. However, a very similar bound also holds in the MSSM and, indeed, in all SUSY models where the Higgs sector is required to remain perturbative up to some high scale [32, 18]. This test would therefore not be very specific. While the failure to detect such a Higgs boson would rule out a very large class of models, including the $E(6)$ models we are considering here, discovery of the Higgs boson may not be sufficient to distinguish between the present model and the MSSM, or even the non-supersymmetric SM. In contrast, the large mass splitting between the light and heavy exotic leptons implies that exotic lepton production should not be confused with the production of charginos and neutralinos predicted by the MSSM. Large invisible branching ratios for light Higgs bosons also clearly indicate the presence of (super)fields beyond those contained in the (MS)SM, unless the mass of the light chargino is close to its present lower bound. Singlet Majoron models [33] can also have light Higgs bosons with large invisible branching ratios [34]. However, in such models the light bosons have sizable $SU(2)$ singlet components, in contrast to the models we are studying here, where the large mass of the Z' boson forces all light Higgs bosons to be predominantly $SU(2)$ doublets. A detailed study of the production and decay of the light Higgs boson(s) should therefore be able to distinguish between the $E(6)$ model and the singlet Majoron model.

Finally, we would like to emphasize that the decisive test we have devised here relies on the versatility of high energy e^+e^- colliders. Clearly the cross sections we are studying are too small to give viable signals at hadron colliders. Moreover, the search for the production of neutral leptons, in particular the associate production of a light and a heavy neutral exotic, plays an important role in our analysis. This would not be feasible at $e\gamma$ or $\gamma\gamma$ colliders, which may also be unable to discover, let alone study, Higgs bosons with large invisible branching ratios. All these colliders will be able to impose constraints on, or – with luck – to discover, some of the new particles predicted by $E(6)$ models. However, only searches at e^+e^- colliders, when combined with cosmological considerations, seem capable of excluding these models completely.

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References

- [1] For a review, see J.L. Hewett and T.G. Rizzo, Phys. Rep. **183**, 193 (1989).
- [2] M. Green, J. Schwarz and E. Witten, *Superstrings* (Oxford University Press, Oxford, England, 1987).
- [3] A. Djouadi, J. Ng and T.G. Rizzo, SLAC PUB-95-6772, hep-ph 9504210, Report of the DPF Study Group on Electroweak Symmetry Breaking and New Physics, Subgroup on New Particles and Interactions; M. Cvetič and S. Godfrey, Carleton Univ. report OCIP-C-95-2, hep-ph 9504216, Report of the DPF Study Group on Electroweak Symmetry Breaking and New Physics, Subgroup on Extra Gauge Bosons.
- [4] M. Drees and S.P. Martin, Univ. Wisconsin report MADPH-879, hep-ph 9504324, Report of the DPF Study Group on Electroweak Symmetry Breaking and New Physics, Subgroup on SUSY Model Building.
- [5] J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, Nucl. Phys. **B276**, 14 (1986); L.E. Ibáñez and J. Mas, Nucl. Phys. **B286**, 107 (1987).
- [6] M. Drees, Nucl. Phys. **B298**, 333 (1988).
- [7] M. Gell-Mann, P. Ramond and S. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p.315; T. Yanagida, in *Workshop on Unified Theory and Baryon Number in the Universe*, eds. A. Sawada and H. Sugawara (KEK, Tsukuba, Japan, 1979).
- [8] M.C. Gonzalez-Garcia and J.W.F. Valle, Phys. Lett. **B240**, 163 (1990).
- [9] J. Ellis, D.V. Nanopoulos, S.T. Petcov and F. Zwirner, Nucl. Phys. **B283**, 93 (1987).
- [10] M. Drees and X.R. Tata, Phys. Lett. **B196**, 65 (1987).
- [11] E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley (New York, 1990).
- [12] S. Weinberg and S. Glashow, Phys. Rev. **D15**, 1958 (1977); E.A. Paschos, Phys. Rev. **D15**, 1966 (1977).
- [13] M. Drees and X.R. Tata, Phys. Rev. Lett. **59**, 528 (1987).
- [14] A. Datta, M. Drees, R.M. Godbole and X.R. Tata, Phys. Rev. **D37**, 1876 (1988).
- [15] E. Cohen, J. Ellis, K. Enqvist and D.V. Nanopoulos, Phys. Lett. **165B**, 76 (1985).
- [16] J.C. Romão, Phys. Lett. **B173**, 309 (1986).
- [17] M. Drees, Phys. Rev. **D35**, 2910 (1987).
- [18] D. Comelli and C. Verzegnassi, Phys. Rev. **D47**, 764 (1993).

- [19] B.A. Campbell, J. Ellis, K. Enqvist, M.K. Gaillard and D.V. Nanopoulos, *Int. J. Mod. Phys.* **A2**, 831 (1987).
- [20] ALEPH collab., D. Decamp et al., *Phys. Rep.* **216**, 253 (1992); L3 collab., M. Acciarri et al., *Phys. Lett.* **B350**, 109 (1995); and references therein.
- [21] D. Schaile, CERN report PPE/94-162, Talk given at the *27th International Conference on High Energy Physics*, Glasgow, Scotland, July 1994.
- [22] ALEPH collab., D. Decamp et al., *Phys. Lett.* **B265**, 475 (1991).
- [23] J. Kamoshita, Y. Okada and M. Tanaka, *Phys. Lett.* **B328**, 67 (1994).
- [24] A. Djouadi, *Int. J. Mod. Phys.* **A10**, 1 (1995).
- [25] M. Drees and K.-I. Hikasa, *Phys. Lett.* **B240**, 455 (1990); erratum, **B262**, 497 (1991).
- [26] See e.g. J.R. Primack, J. Holtzman, A. Klypin and D.O. Caldwell, *Phys. Rev. Lett.* **74**, 2160 (1995), and references therein.
- [27] K. Griest and D. Seckel, *Phys. Rev.* **D43**, 3191 (1991).
- [28] Particle Data Group, L. Montanet et al., *Phys. Rev.* **D50**, 1173 (1994).
- [29] R. Arnowitt and P. Nath, *Phys. Lett.* **B299**, 58 (1993); erratum, **B307**, 403 (1993); J.L. Lopez, D.V. Nanopoulos and K. Yuan, *Phys. Rev.* **D48**, 2766 (1993).
- [30] O.J.P. Eboli, M.C. Gonzalez-Garcia, A. Lopez-Fernandez, S.F. Novaes and J.W.F. Valle, *Nucl. Phys.* **B421**, 65 (1994).
- [31] K. Olive and G. Steigman, *Phys. Lett.* **B354**, 357 (1995).
- [32] G.L. Kane, C. Kolda and J. Wells, *Phys. Rev. Lett.* **70**, 2686 (1993); J.R. Espinosa and M. Quiros, *Phys. Lett.* **B302**, 51 (1993); J.R. Espinosa, DESY report 95-037, hep-ph 9503255.
- [33] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Phys. Lett.* **98B**, 265 (1980).
- [34] A.S. Joshipura and S.D. Rindani, *Phys. Rev. Lett.* **69**, 3269 (1992).

Figure Captions

Fig. 1 The fraction of parameter sets that survive certain constraints is shown as a function of the mass $m_{L_1^0}$ of the lightest neutral exotic lepton, for $\tan\beta = 1.2$ (a) and 5 (b). The fraction is relative to the total number of sets generated in a given bin. For the dotted histograms only the current, LEP1 constraints (25),(26) have been imposed, while the solid histograms show the fraction of parameter sets that in addition satisfy $\Omega h^2 < 1$. Recall that we only include $f\bar{f}$ final states in the calculation of the relic density; this under-estimates the annihilation cross sections, i.e. over-estimates the relic density, for $m_{L_1^0} > m_W$. When choosing the parameters of the neutral lepton mass matrix (3), we have required the charged exotic leptons to be heavier than 40 GeV.

Fig. 2 The fraction of parameter sets that survive present LEP1 constraints (dotted), exotic lepton searches at LEP2 (dashed), and the relic density bound $\Omega h^2 < 1$ (solid). We have assumed that LEP2 will be able to detect pair production of neutral exotic leptons if the total cross section for final states where at least one leptons is more than 5 GeV heavier than L_1^0 exceeds 20 fb. For reasons explained in the text, we have imposed the restrictions (46) on the parameters of the neutral lepton mass matrix (3), which implies that the 6 neutral Majorana leptons pair up to form 3 Dirac states. We have required the charged exotic leptons to be heavier than 80 GeV here. Results are for $\tan\beta = 1.2$ (a) and 5 (b).

Fig. 3 As in Fig. 2, except that we have required the charged exotic leptons to be heavier than 240 GeV, and the dashed histograms show the fraction of parameter sets that would not lead to an observable signal for the pair production of exotic leptons at a 500 GeV e^+e^- collider (“NLC1”). Here a signal is considered to be observable if the total cross section for the production of neutral exotic leptons, with one final state lepton being at least 5 GeV heavier than L_1^0 , exceeds 0.5 fb.

Fig. 4 As in Fig. 2, except that we have required the charged exotic leptons to be heavier than 700 GeV, and the dashed histograms show the fraction of parameter sets that would not lead to an observable signal for the pair production of exotic leptons at a 1.5 TeV e^+e^- collider (“NLC3”). Here a signal is considered to be observable if the total cross section for the production of neutral exotic leptons, with one final state lepton being at least 5 GeV heavier than L_1^0 , exceeds 0.05 fb. As explained in the text, all surviving parameter sets (solid histograms) predict a large invisible branching ratio for at least one light neutral Higgs boson, which allows to test the model decisively.

Fig. 5 The maximal value of the mass $m_{L_1^0}$ of the lightest neutral exotic lepton, computed from the ansatz (47) with $\lambda_{\max} = 0.85$, is shown as a function of $\tan\beta$. We have required $m_{L_i^\pm} > m_{L^+, \min}$, and $|m_{L_1^0}| + |m_{L_3^0}| > 2m_{L^+, \min}$, for different values of $m_{L^+, \min}$ as indicated.

Fig. 6 The minimal masses of the Higgs bosons of the model, as determined from present LEP1 constraints (28), is shown as a function of $\tan\beta$. The solid, dashed and dotted curves show the smallest allowed mass of the lightest neutral Higgs scalar, of the pseudoscalar, and the minimal sum of the masses of the lightest scalar and pseudoscalar Higgs bosons, respectively. The minimum of the sum is significantly larger than the sum of the minima, indicating that both Higgs masses cannot be minimized simultaneously.